ELENE6909

Elevator Scheduling and Performance Analysis

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AGENDA

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- **Performance analysis of an elevator system during up-peak**
  one elevator; up-peak period

- Decision-theoretic group elevator scheduling
  multiple elevators; any situations

Content of the paper

- Description of model
- Number of passengers in the elevator system
- Elevator round-trip time
- Passenger waiting time at the lobby
- Passenger ride time
- Passenger journey time

Description of model

- Parameters

N	number of floors above the lobby
t_r	run time of an elevator for one floor at the nominal speed
t_l	stop time of the elevator at the lobby
t_s	stop time at the other floors
c	elevator capacity

Description of model

- Assumptions

- Passengers arrive at the lobby according to a Poisson process with rate λ
- No passengers arrive at the other floors
- A single elevator serves by First-Come First-Served (FCFS)
- The destination floor is uniformly distributed among all floors
- The elevator stays idle at the highest destination floor until the first passenger arrives at the lobby

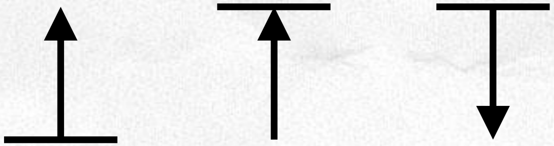
Description of model

- Continuous time to discrete time

Departure moment from the lobby

Arrival moment at the highest destination floor

Departure moment from the highest destination floor back to the lobby



Number of passengers in the elevator system

L_n: # passengers F_n: location of the elevator S_n: # stops between (n-1) and nth states

DTMC: $\{(L_n, F_n), n = 1, 2, \ldots\}$

State space: $\{(i, j), i = 0, 1, \ldots, j = 0, 1, \ldots, N, i + j > 0\}$

Define:
$$G_i(l, m) = \underbrace{\frac{1}{N^i}}_{\text{total \# conditions}} \underbrace{\binom{l-1}{m-1}}_{\text{\# stop choice}} \underbrace{\sum_{\substack{g_1+\cdots+g_m=i \\ 1\le g_1,\ldots,g_m \le i-m+1}} \prod_{e=1}^{m} \binom{i - \sum_{a=1}^{e-1} g_a}{g_e}}_{\text{\# conditions for each stop choice}},$$

$$1 \le i \le c, 1 \le l \le N, 1 \le m \le (i, l)^-$$

Number of passengers in the elevator system

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Transition probability given there are *m* stops between the two states:

$$Q_{(i,j),(k,l)}(m) \equiv P\{L_n = k, F_n = l, S_n = m | L_{n-1} = i, F_{n-1} = j\},$$
$$i, k = 0, 1, \ldots, j, l, m = 0, 1, \ldots, N, i + j > 0, \quad k + l > 0.$$

$$= \begin{cases} G_{(i,c)^-}(l,m)[\lambda(lt_r + mt_s)]^{k-(i-c)^+} \mathrm{e}^{-\lambda(lt_r + mt_s)} / (k - (i-c)^+)! \\ \quad \text{if } i \geq 1, j = 0, k \geq (i-c)^+, 1 \leq l \leq N, 1 \leq m \leq (i,l,c)^-, \\ [\lambda(t_l + jt_r)]^{k-i} \mathrm{e}^{-\lambda(t_l + jt_r)} / (k-i)! \quad \text{if } i \geq 1, 1 \leq j \leq N, k \geq i, l = 0, m = 1, \\ 1 \quad \text{if } i = 0, 1 \leq j \leq N, k = 1, l = j, m = 0, \\ 0 \quad \text{otherwise,} \end{cases}$$

Number of passengers in the elevator system

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Transition probability given there are m

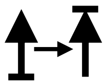
$$P\left(l,k,m \mid i, j=0\right) = P\left(l,m \mid i, j=0\right) \times P\left(k \mid l,m,i, j=0\right)$$

$$P\left(\Delta passengers = x\right) = e^{-\lambda t}\frac{\left(\lambda t\right)^{x}}{x!}$$

$$Q_{(i,j),(k,l)}(m) \equiv P\{L_n = k, F_n = l, S_n = \ldots$$

$$i,k = 0,1,\ldots,j,l,m = 0,1,\ldots,N, i+j>0, \quad k+l>0.$$

$$= \begin{cases} \boxed{\begin{array}{l} G_{(i,c)^-}(l,m)[\lambda(lt_r+mt_s)]^{k-(i-c)^+}e^{-\lambda(lt_r+mt_s)}/(k-(i-c)^+)! \\ \quad \text{if } i \geq 1, j=0, k \geq (i-c)^+, 1 \leq l \leq N, 1 \leq m \leq (i,l,c)^-, \end{array}} \\ [\lambda(t_l+jt_r)]^{k-i}e^{-\lambda(t_l+jt_r)}/(k-i)! \quad \text{if } i \geq 1, 1 \leq j \leq N, k \geq i, l=0, m=1, \\ 1 \quad \text{if } i=0, 1 \leq j \leq N, k=1, l=j, m=0, \\ 0 \quad \text{otherwise,} \end{cases}$$



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$$i, k = 0, 1, \ldots, j, l, m = 0, 1, \ldots,$$

$$P(k) = P(\Delta passengers = x) = e^{-\lambda t}\frac{(\lambda t)^x}{x!}$$

$$= \begin{cases} G_{(i,c)^-}(l,m)[\lambda(lt_r + mt_s)]^{k-(i-c)^+} e^{-\lambda(lt_r+mt_s)}/(k-(i-c)^+)!, \\ \quad \text{if } i > 1, j = 0, k > (i-c)^+, 1 < l < N, 1 < m < (i,l,c)^-, \\ \boxed{[\lambda(t_l + jt_r)]^{k-i} e^{-\lambda(t_l+jt_r)}/(k-i)! \quad \text{if } i \geq 1, 1 \leq j \leq N, k \geq i, l = 0, m = 1,} \\ 1 \quad \text{if } i = 0, 1 \leq j \leq N, k = 1, l = j, m = 0, \\ 0 \quad \text{otherwise,} \end{cases}$$

Number of passengers in the elevator system

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Transition probability given there are m stops between the two states:

$$
\begin{aligned}
Q_{(i,j),(k,l)}(m) \equiv\ & P\{L_n = k, F_n = l, S_n = m \mid L_{n-1} = i, F_{n-1} = j\}, \\
& i, k = 0, 1, \ldots, j, l, m = 0, 1, \ldots, N, i + j > 0, \quad k + l > 0.
\end{aligned}
$$

$$
= \begin{cases}
G_{(i,c)^-}(l, m)[\lambda(lt_r + mt_s)]^{k-(i-c)^+} e^{-\lambda(lt_r + mt_s)}/(k - (i-c)^+)! \\
\quad \text{if } i \geq 1, j = 0, k \geq (i-c)^+, 1 \leq l \leq N, 1 \leq m \leq (i, l, c)^-, \\
[\lambda(t_l + jt_r)]^{k-i} e^{-\lambda(t_l + jt_r)}/(k-i)! \quad \text{if } i \geq 1, 1 \leq j \leq N, k \geq i, l = 0, m = 1, \\
\boxed{1 \quad \text{if } i = 0, 1 \leq j \leq N, k = 1, l = j, m = 0,} \\
0 \quad \text{otherwise,}
\end{cases}
$$

Number of passengers in the elevator system

L_n: # passengers F_n: location of the elevator S_n: # stops between (n-1) and nth states

Transition matrix: $P = \{P_{(i,j),(k,l)}\}$:

$$P_{(i,j),(k,l)} = \sum_{m=0}^{(i,l,c)^-} Q_{(i,j),(k,l)}(m)$$

Steady-state distribution: $\{\pi_{i,j}\}$ → M/G/1 queue

Elevator round-trip time (R)

Distribution function: $R(x) \equiv P\{R \le x\}$

$$P(i) = \pi_{i,0} / \sum_{j=1}^{\infty} \pi_{j,0},\ i = 1, 2, \ldots$$

$$R_i^d(x) = \sum_{k=1}^{\infty} \sum_{l=1}^{N} \sum_{m=1}^{\lceil (x-2lt_r-t_l)/t_s \rceil} Q_{(i,0),(k,l)}(m),$$

The time the elevator waiting at the highest destination floor = 0

$$R_i^c(x) = \sum_{l=1}^{N} \sum_{m=1}^{\lceil (x-2lt_r-t_l)/t_s \rceil} Q_{(i,0),(0,l)}(m) \underbrace{\left[1 - e^{-\lambda(x-mt_s-2lt_r-t_l)}\right]}$$

The time the elevator waiting at the highest destination floor > 0

$$R_{c+1}^c(x) = 0,\ \text{i.e.},\ R_{c+1}(x) = R_{c+1}^d(x)$$

P [waiting time ≤ ($x-mt_s-2lt_r-t_l$)]

| Performance analysis of an elevator system during up-peak |

Elevator round-trip time (*R*)

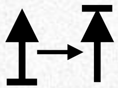
Distribution function: $R(x) \equiv P\{R \le x\}$

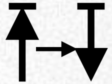
$$R_i(x) = P\left(R_i^d(x) \cup R_i^c(x)\right) = R_i^d(x) + R_i^c(x) \quad i = 1, \ldots, c+1$$

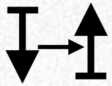
$$R(x) = \sum_i P(i) \cdot R_{(i,c+1)^-}(x)$$

Passenger waiting time at the lobby (*W*)

Time period between a state (*i, j*) and the next state $X_{i,j}$:

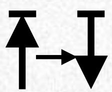
 $i > 0, j = 0$ $P\left(X_{ij} = lt_r + mt_s\right) = G_i\left(l,m\right)$ $\sum_{l=1}^{N} \sum_{m=1}^{(i,l,c)^-} (lt_r + mt_s) G_{(i,c)^-}(l, m)$

 $i = 0, j > 0$ $P\left(X_{ij}\right) \sim Exp\left(\lambda\right)$ $\xrightarrow{E\left\{X_{ij}\right\}}$ $1/\lambda$

 $i > 0, j > 0$ $P\left(X_{ij} = jt_r + t_l\right) = 1$ $jt_r + t_l$

Passenger waiting time at the lobby (*W*)

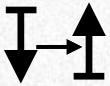
Conditional waiting time $W_{i,j}$ (passenger arrives during the period $X_{i,j}$):



$i = 0, j > 0$ $\qquad$ $W^*_{0,j}(s) = e^{-s(jt_r + t_l)}$ $\qquad$ LST (Laplace-Stieljes Transform)

Passenger waiting time at the lobby (*W*)

Conditional waiting time $W_{i,j}$ (passenger arrives during the period $X_{i,j}$):



$i > 0, j > 0$

$$\longrightarrow \boxed{y} \longrightarrow \boxed{R^*_{c+1}} \longrightarrow$$

$$E\left[e^{-sW_{i,j}} \mid \text{arrival at } y, \alpha = n\right] = e^{-sy}\left[R^*_{c+1}(s)\right]^{\lceil (i+n)/c \rceil}$$

$$\xrightarrow{\sum_{n=0}^{\infty}(\cdot)P(\alpha = n)} E\left[W^*_{ij} \mid y\right] \xrightarrow{\int_y (\cdot)P(y)\,dy} W^*_{ij}$$

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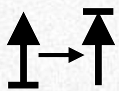
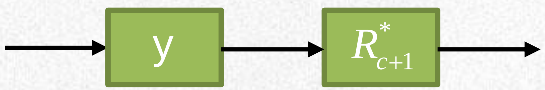


$i > 0, j = 0$



$$E\left[e^{-sW_{i,0}} \middle| X_{i,0} = lt_r + mt_s,\ \text{arrival at } y, \alpha = n\right] = e^{-s(y+lt_r+t_l)}\left[R^*_{c+1}(s)\right]^{\lceil \{i-(i,c)^- + n\}/c \rceil}$$

$$\xrightarrow{\sum_{n=0}^{\infty}(\bullet)P(\alpha=n)} E\left[W^*_{ij} \mid X_{i,0} = lt_r + mt_s, y\right] \xrightarrow{\int_y(\bullet)P(y)dy} E\left[W^*_{ij} \mid X_{i,0} = lt_r + mt_s\right] \xrightarrow{\sum_{l,m}(\bullet)P(l,m)} W^*_{ij}$$

Passenger waiting time at the lobby (*W*)

Waiting time *W*:

$$W^* = \sum_{i,j} W_{ij}^* P\left(arrive\ during\ X_{ij}\right)$$

$$= \sum_{i,j} W_{ij}^* \frac{\pi_{ij} \cdot E\left\{X_{ij}\right\}}{\sum_{k,l} \pi_{kl} \cdot E\left\{X_{kl}\right\}}$$

$$= \sum_{i,j} \pi_{i,j}\eta_{i,j}W_{i,j}^*(s) \Big/ \sum_{k,l} \pi_{k,l}\eta_{k,l} \xrightarrow{\text{ILST}} W$$

Passenger ride time (*T*) (step in → step out)

Calculation steps:

M: # passengers in the elevator   *l*: destination floor   *m*: # stops before floor *l*

$$P(M=i)$$
$$P(T=lt_r+mt_s \mid M=i)$$

$$\longrightarrow \quad P\{T=lt_r+mt_s\}=\sum_{i=m+1}^{c} P\{M=i\}P\{T=lt_r+mt_s|M=i\}$$

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Passenger journey time (*J*) (arrival → step out)

Similar with the way of calculating waiting time

$$i > 0, j = 0 \qquad \longrightarrow \boxed{y} \longrightarrow \boxed{lt_r + t_l} \longrightarrow \boxed{R^*_{c+1}} \longrightarrow \boxed{T^*_k} \longrightarrow J^*_{i,0}$$

$$i = 0, j > 0 \qquad \longrightarrow \boxed{jt_r + t_l} \longrightarrow \boxed{T^*_k} \longrightarrow J^*_{0,j}$$

$$i > 0, j > 0 \qquad \longrightarrow \boxed{y} \longrightarrow \boxed{R^*_{c+1}} \longrightarrow \boxed{T^*_k} \longrightarrow J^*_{i,j}$$

$$\longrightarrow J^* = \sum_{i,j} J^*_{ij} P\left(arrive\ during\ X_{ij}\right) = \sum_{i,j} J^*_{ij} \frac{\pi_{ij} \cdot E\left\{X_{ij}\right\}}{\sum_{k,l} \pi_{kl} \cdot E\left\{X_{kl}\right\}} \xrightarrow{\text{ILST}} J$$

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Content of the paper

- Assumptions
- Optimization criterion
- Effect of uncertainty in destination
- Dynamic Programming algorithm
- Extension of the algorithm

Assumptions

- Future arrivals are not considered into decisions
- Destination floors are uniformly distributed
- The number of people waiting on each floor is known
- Keep moving in the current direction until all passengers in this direction are picked up and delivered
- Previous assignments are never changed
- Each elevator has infinite capacity

Optimization criterion

- Minimize the total remaining waiting time of all currently waiting passengers

  remaining waiting time: from current moment to the moment that the person is picked up

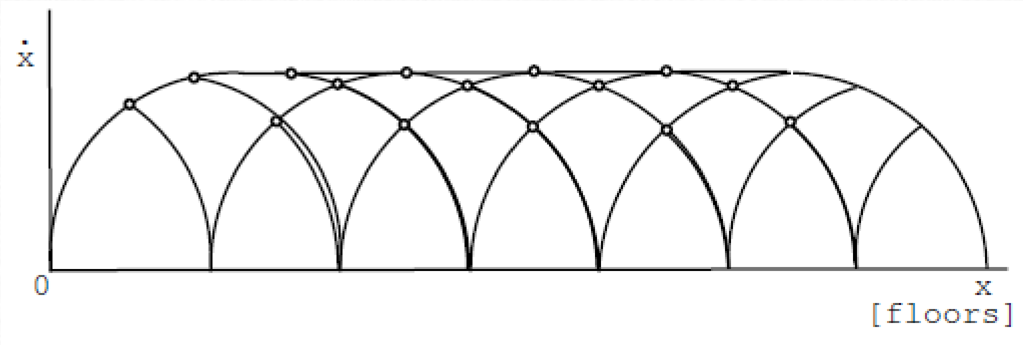
- W_i: the expected remaining waiting time after assigning the new call to elevator i
- Want to find i s.t. W_i is minimum

Effect of uncertainty in destination

- The expectation of the waiting time is taken with respect to the uncertainty in the destinations of passengers who are not yet to be picked up by the elevator (their destinations are unknown)
- The expectation can be obtained by computing the waiting time along each path, weighting these times by the probability of the respective path
- Suppose N_p passengers and N_f floors, then the complexity is $O\left(N_f^{N_p}\right)$

Dynamic Programming algorithm ESA-DP

- To reduce the complexity from $O\left(N_f^{N_p}\right)$ to $O\left(N_f N_p\right)$
- Branching points: last possible location at which an elevator should start decelerating if it is to stop at a particular floor in its direction of motion.



Dynamic Programming algorithm ESA-DP

- DTMC: (*f, d, v, n*)
- *f:* the floor at which the elevator will stop if it starts decelerating at that branching point
- *d:* the elevator's current direction (up or down)
- *v:* the elevator's velocity at current branching point (0 to N_v-1)
- *n*: the number of newly boarded passengers
- Calculation steps: calculate the waiting time between each two branching points and get the expected waiting time at each state (branching point)

Dynamic Programming algorithm ESA-DP

- Example

  - The elevator will stop at floor 13 if it decelerates now

  - It has been scheduled to pick up a passenger at floor 7

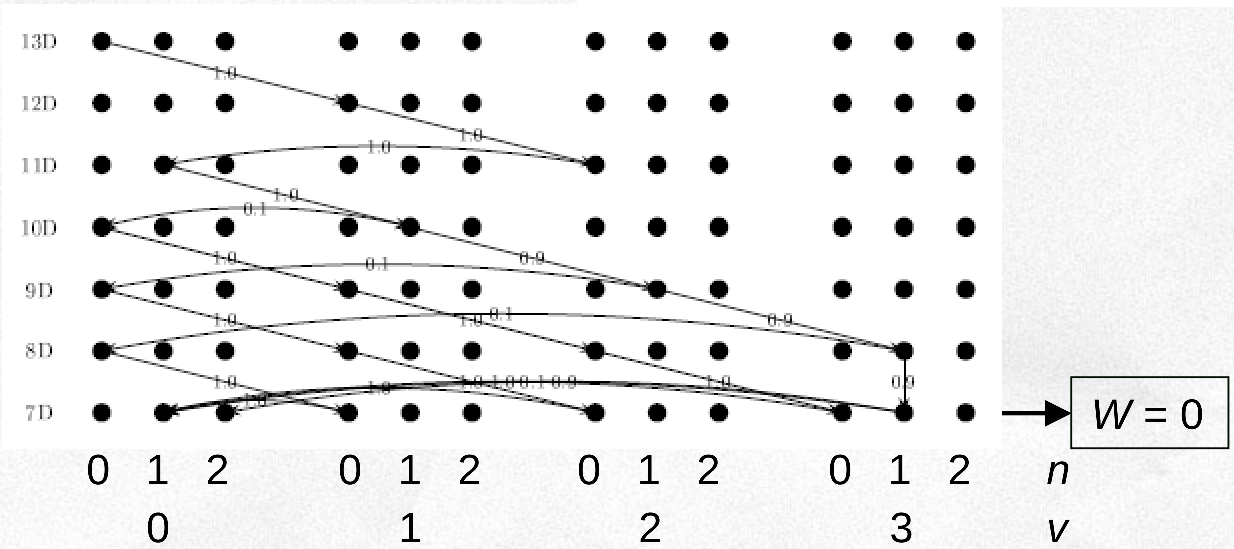
  - The problem is considering whether this elevator should also pick up a new passenger at floor 11

Dynamic Programming algorithm ESA-DP

- Example

$$Pr(x, n, k) = \frac{n!}{(n-x)!x!} \frac{(k-1)^{n-x}}{k^n}$$

f/d

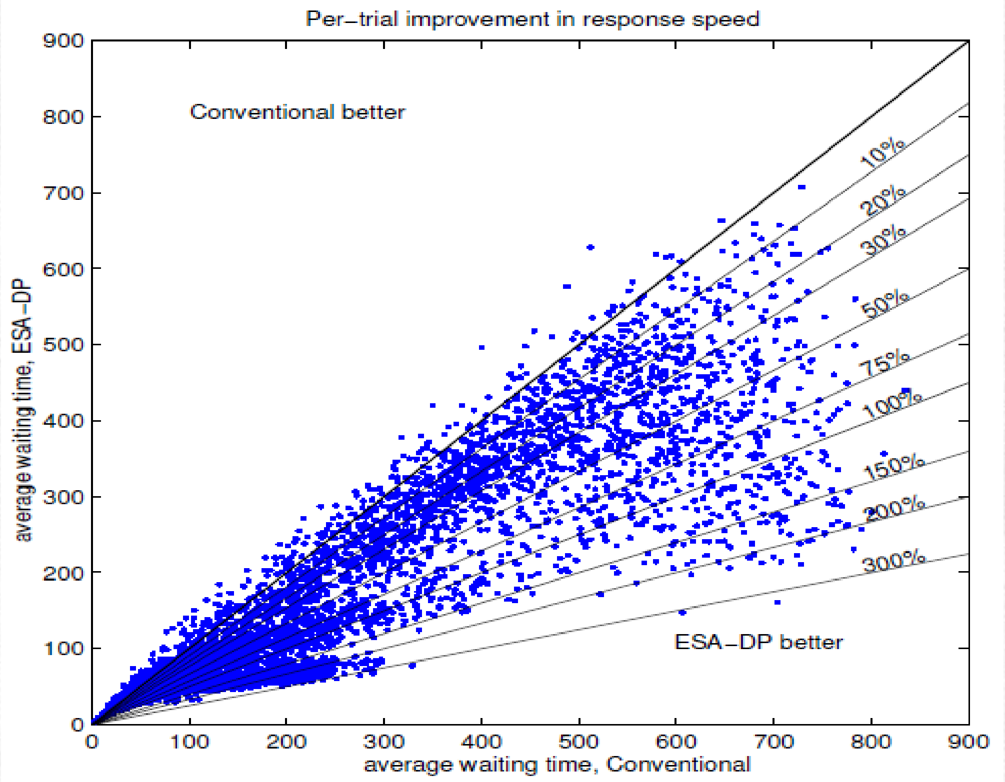


Dynamic Programming algorithm ESA-DP

- The algorithm iteratively computes the expected remaining waiting time of each state in the diagram that can be visited by the elevator.
- After all expected remaining waiting times are computed, the overall expected remaining waiting time is just the value of the initial state
- Compute from left to right, from bottom row to the first row

Dynamic Programming algorithm ESA-DP

- Simulation



Extension of the algorithm

- The number of people at one floor is unknown
  - use other ways to obtain the number
  - use the expected number instead of real number $E[X] = \lambda t$
- Non-uniform destination distribution
  - change the function of $P(x,n,k)$
- Finite elevator capacity
  - divide the states into sub-states: number of people in the elevator and number of people who will get off at that floor

References

[1] Lee, Yutae, et al. "Performance analysis of an elevator system during up-peak." *Mathematical and Computer Modelling* 49.3 (2009): 423-431.

[2] Nikovski, Daniel, and Matthew Brand. "Decision-Theoretic Group Elevator Scheduling." *ICAPS*. Vol. 3. 2003.

[3] Nagatani, Takashi. "Complex behavior of elevators in peak traffic." *Physica A: Statistical Mechanics and its Applications* 326.3 (2003): 556-566.

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