1. Consider the Chernoff bound for the summation of \( n \) iid random variables \( X_1, \ldots, X_n \), i.e.,
\[
P\left( \sum_{i=1}^{n} X_i \geq na \right) \leq e^{-nh(a)}.
\]
Prove that \( h(a) > 0 \) if \( a > E[X_1] \) and \( E[e^{\theta X_1}] < \infty \) over \( \theta \in [0, b] \) for some \( b > 0 \).

2. Find a Chernoff bound for the summation of \( n \) iid random variables \( X_1, \ldots, X_n \) in the form
\[
P\left( \sum_{i=1}^{n} X_i \leq na \right) \leq e^{-n\tilde{h}(a)},
\]
and identify \( \tilde{h}(a) \).

3. Let \( X \) be the number of heads in a sequence of \( n \) independent fair coin flips, and \( \mu = E[X] = \frac{n}{2} \). Apply the Chernoff bound to show that for any \( \delta > 0 \),
\[
P(|X - \mu| > \delta \mu) \leq 2 \exp\left(-\frac{\mu \delta^2}{3}\right).
\]
Then use this bound to show that the deviation from the mean \( \mu \) is of the order \( O(\sqrt{n \log n}) \).

4. Consider the exponentially tilted distribution \( \{\hat{p}_k(s)\} \), \( s \in \mathbb{R} \),
\[
\hat{p}_k(s) = p_k s^k \frac{\phi^k}{\phi(s)}, \quad k = 0, 1, \ldots,
\]
where \( \phi(s) \) is the generating function of \( \{p_k\} \). What is the mean of \( \{\hat{p}_k(s)\} \)?

5. Geometric branching process: Consider a Galton-Watson branching process with geometric offspring distribution with parameter \( p \), i.e. \( P(\xi = k) = p^k (1 - p) \).
   (i) Compute the probability of extinction occurring in generation \( n \) (using generating functions).
   (ii) Give a general expression for the probability of extinction.
   (iii) Derive an expression for the generating function of
\[
W = \lim_{n \to \infty} \frac{X_n}{(E(\xi))^n},
\]
where \( X_n \) is the number of individuals in generation \( n \).