

Network Algorithms and Dynamics

Homework 3

Due: 03/09/2017

1. Suppose $\{X(t), t \geq 0\}$ is a martingale. Show that $\{e^{\theta X(t)}, t \geq 0\}$ is a nonnegative submartingale ($\theta > 0$). Recall that we used this result to find the concentration for the poisson process by applying Doob's inequality.
2. Consider a SIER epidemic process as follows. Every infected (I) individual interacts with another individual chosen uniformly at random at rate β . Population size is n . When a susceptible (S) individual becomes exposed (E) to an infected individual, it takes an exponentially distributed amount of time with parameter γ to show the symptoms at which point he/she becomes infected. Each infected individual has an infectious period exponentially distributed with parameter δ during which he/she can infect others after which he/she becomes removed (R). First describe the system using a continuous-time Markov chain by describing the transition rates between various states. Second, write a set of differential (mean-field) equations for the system. In what sense the differential equations provide a good approximation to the Markov process?
3. Show that under the small-world network model, with n nodes and for any $\alpha > 0$, the maximum degree is $O(\log n)$ with high probability.
4. (Refreshing your memory about Poisson process) Jobs arrive at a data center according to a Poisson process with rate λ (jobs/hour). Suppose two jobs arrived during the first hour. What is the probability that (a) both arrived during the first 20 minutes? (b) at least one arrived during the first 20 minutes?