Network Algorithms and Dynamics

Homework 6 Due: 05/02/2016

1. In the class we presented two definitions for a submodular function. Show that two definitions are equivalent, i.e., if a function f satisfies definition 1, it satisfies definition 2 and vice versa.

2. (Facility Location, or Sensor Placement) Suppose we wish to select, out of a set $V = \{1, \cdots, n\}$, some locations to open up facilities (place sensors) in order to serve a collection of m customers. If we open up a facility at location j, then it provides service of value M_{ij} to customer i, where $M \in \mathbb{R}_+{}^{m \times n}$ is given. Suppose each customer chooses the facility with highest value.

   (a) Show that the total value provided to all customers can be modeled by a set function which is monotone submodular.

   (b)Describe a greedy algorithm to select k facility locations and provide its performance bound with respect to the optimal placement.

3. (Overview of random variable generation) Let X be a random variable whose distribution can be described by the cumulative distribution function (CDF) F. We want to generate values of X which are distributed according to this distribution. As we mentioned in the class, one way to do this is to generate random number U uniformly distributed on $[0, 1]$ and then generate X as $X = F^{-1}(U)$. Show that this generates the true CDF F. How should we interpret the inversion F^{-1} if X is not a continuous random variable to ensure the method still works?

4. (Variants of the join-the-shortest-queue policy) Consider a system with 2 servers and job arrival rate λ (Poisson process). Each server has an individual buffer however servers work at different speeds: the processing time of a job at server i is exponentially distributed with mean $1/\mu_i$, $i = 1, 2$. Propose a scheduling scheme to minimize the average delay for serving jobs in the system. You do not need to prove the optimality however you must show that the system is stable under your proposed scheme, i.e., for any $\lambda < \mu_1 + \mu_2$, the queues remain bounded (the system Markov chain is positive recurrent).